Isoscaling in statistical fragment emission in an extended compound nucleus model

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Abstract

Based on an extended compound nucleus model, isospin effects in statistical fragment emission from excited nuclear systems are investigated. An experimentally observed scaling behavior of the ratio of isotope yields $Y_i(N,Z)$ from two similar emitting sources with different neutron-to-proton ratios is predicted theoretically, i.e., the relationship of $Y_2/Y_1 \propto exp(\alpha N + \beta Z)$ is demonstrated. The symmetry energy coefficient C_{sym} extracted from the simulation results is ~ 27 MeV which is consistent with realistic theoretical estimates and recent experimental data. The influence of the surface entropy on the isoscaling behavior is discussed in detail. It is found that although the surface entropy increases the numercial values of isoscaling parameters α and β, it does not affect the isoscaling behavior qualitatively and has only a minor effect on the extracted symmetry energy coefficient.

Key words: Isoscaling, Compound nucleus, Fragment emission
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1 INTRODUCTION

The search for a nuclear equation of state (EOS) has been one of the dominant factors driving interest in heavy-ion collisions at intermediate and high energies, ever since such beams have become available. Exploration of the isospin dependence of the EOS is an inseparable part of the task. Originally, the latter studies were confined to use stable projectile beams with only a moderately wide range of isotopes between neutron-poor and neutron-rich nuclei.

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However, exotic secondary beams of projectile nuclei with extreme neutron-to-proton ratios that have become available recently offer new and intriguing opportunities to study isospin physics in heavy-ion collisions [1,2,3]. These studies have already resulted in the discovery of several interesting isospin-related systematics [4,5,6,7], prompting further efforts both, in experiments and in theoretical modeling. In particular, a so-called isoscaling analysis of light fragment emission in the decay of very hot nuclear systems produced in heavy-ion collisions has attracted attention as a possible tool for deducing the nuclear symmetry energy from the relative fragment yields [7,8,9,11,10].

For strongly damped collisions fragment isotopic yields were found to follow a "Q_{gg} systematics" [12,13,14]. Isotopic scaling, termed "isoscaling" [7,8,9], is observed for other types of reactions such as evaporation [7,15], fission [16,17], projectile fragmentation [18,19], and multifragmentation [7,5,6]. This scaling law refers to a general exponential relation between the ratios of yields $Y_i(N,Z)$ for fragments (N,Z) emitted from systems which differ only in their isospin or $(\frac{N_s}{Z_s})_i$. In particular, if two reactions lead to primary systems $i=1$ and 2 having approximately the same temperature but different isospin, the ratio $R_{21}(N,Z)$ of the experimental yields of a given fragment (N,Z) emitted from these systems exhibits an exponential dependence on the fragment neutron number N and atomic number Z of the form,

$$R_{21}(N,Z) = \frac{Y_2(N,Z)}{Y_1(N,Z)} = Cexp(\alpha N + \beta Z). \tag{1}$$

Here, α and β are the isoscaling parameters and C is an overall normalization constant.

On the theoretical side, isoscaling has been extensively examined in the antisymmetrical molecular dynamics model [20], a Boltzmann-Uehling-Uhlenbeck model [5], a lattice gas model [21,22], the expanding evaporating source model [23] and in statistical multifragmentation models [8,9,11,24]. In the present paper it will be shown how the essential features of isoscaling observed in experiment are related to the nuclear symmetry energy in an extended compound nucleus (ECN) model. This model is closely related to that known from fission studies, and its central notion is a relatively high entropy (particularly at moderately high excitation energies) associated with the diffuse nuclear surface region (as opposed to bulk matter). Application of this ECN model reveals a new mechanism of nuclear fragmentation caused by the softening of the diffuse nuclear surface at high excitations [25]. The ECN model is equivalent to conventional compound nucleus models at low excitation energy as represented, e.g., by GEMINI [26], but greatly extends the validity of the compound nucleus concept towards high excitations [27]. Therefore, the ECN model provides a unified description of statistical emission of light particles and fragments in a wide range of excitation energies. While the present model

is still somewhat schematic, it is based on a physically transparent picture and thus provides direct insight into the phenomena of interest.

2 THEORETICAL FRAMEWORK

The probability p of emitting a fragment from an equilibrated compound nucleus (CN) is evaluated using the Weisskopf formalism [28]:

$$p \propto e^{\Delta S} = e^{S_{saddle} - S_{eq}}, \tag{2}$$

where S_{eq} and S_{saddle} are the entropies for the equilibrated CN and a saddle-point configuration of touching spheres, respectively. Within the Fermi gas model, the entropies for the two configurations can be calculated as

$$S_{eq} = 2\sqrt{a_A E^*_{tot}}, \tag{3}$$

and

$$S_{saddle} = S_{res} + S_{frag} = 2\sqrt{(a_{res} + a_{frag}) E^{*th}_{saddle}}. \tag{4}$$

In Eqs. (3) and (4), a_A, a_{res} and a_{frag} are the level density parameters of the CN system at equilibrium, of the residue, and of the fragment, respectively. The part S_S of the entropy S of the system, associated with the diffuse surface domain, has been found [29] to have a pronounced effect on the fragment emission probability. One has

$$S = S_V + S_S, \tag{5}$$

where S_V is the entropy of bulk matter. Consequently, the level density parameter a includes volume and surface terms [30],

$$a = a_V + a_S = \alpha_V A + \alpha_S A^{2/3} F_2, \tag{6}$$

Here A is the atomic number, F_2 is the surface area relative to a spherical shape. E^{*th}_{saddle} in Eq.(4) is the thermal excitation energy of the system in the saddle-point configuration. The latter quantity is calculated as

$$E^{*th}_{saddle} = E^*_{tot} - V_{saddle}, \tag{7}$$

where V_{saddle} is the collective saddle-point energy.

For the level density parameter a, the parametrization of Tõke and Swiatecki [30] was employed with $\alpha_V = 1/14.6$ MeV^{-1} and $\alpha_S = 4/14.6$ MeV^{-1}. The calculations assume saddle-point shapes to be represented by two touching spheres. V_{saddle} is the difference in deformation energies for the saddle-point shape and equilibrium-state shape. It contains contributions from Coulomb, volume, and surface energies. Volume and surface energies depend on the system isospin I (defined as $(N-Z)/A$) in a functional form $-\alpha_v(1-\kappa_v I^2)A$ and $\alpha_s(1-\kappa_s I^2)A^{2/3}$, respectively. The parameters are taken from [31] as $\alpha_s = 21.13, \kappa_s = 2.3, \alpha_v = 15.9937, \kappa_v = 1.927$. The nuclear temperature can be obtained from the commonly used Fermi-gas model relationship between the temperature T and the excitation energy of the system E^*_{tot}:

$$T = \sqrt{\frac{E^*_{tot}}{a}}. \tag{8}$$

The present calculations do not account for the effects of an expansion of the CN prior to its decay, so the nuclear density is that of the ground state. Therefore the additional influence of the nuclear matter density on the extraction of symmetry energy coefficient is not studied here. The present work concentrates on the temperature dependence of the relation between symmetry energy and isoscaling parameters.

3 RESULTS AND DISCUSSION

In this work several pairs of equilibrated CN sources with proton number $Z_s = 75$ and mass numbers $A_s = 165$, 175, 185, and 195, are considered at initial excitation energies of $E^*_{tot}/\text{A} = 2$, 3, 4, 5, 6, and 7 MeV/nucleon. As the ratio $R_{21}(N,Z)$ is insensitive to sequential decay, it carries information on the original excited fragments prior to their decay [18]. In particular, it has been found that the values of α and β are not much affected by sequential decay of the primary fragments [9]. These observations justify the neglect of sequential decay in the following analysis.

The yield ratio $R_{21}(N,Z)$ is constructed using the convention that index 2 refers to the neutron-rich system and index 1 to the neutron-poor one. The observation summarized in Eq. (1) shows that experimental yield values of $ln(R_{21}(N,Z))$ plotted *vs.* N (Z fixed) or Z (N fixed), produce straight lines. This feature also emerges from the present MCN model. As a demonstration, Fig. 1 shows the theoretical yield ratios $ln(R_{21}(N,Z))$ plotted *vs.* fragment

neutron number N, for individually values of $Z = 6 - 9$ (top panel), and *vs.* proton number Z, for individually values of $N = 6 - 9$ (bottom panel). The ratios have been calculated for the two source pairs ($A_2 = 175$, $A_1 = 165$) and ($A_2 = 185$, $A_1 = 165$) at an excitation energy of $E^*_{tot}/A = 5$ MeV/nucleon. One can see from Fig.1 that isotope ratios of the same element Z, or isotone ratios of the same N, tend to lie on a logarithmic straight line. The solid lines represent linear fits to each series of Z isotopes and each series of N isotones, respectively. It is obvious that these fit lines are nearly parallel to each other. The positive slope in the upper panel of Fig.1 illustrates that neutron-rich fragments are more easily produced from the more neutron-rich sources, as can be expected. Analogously, the negative slope in the bottom panel of Fig.1 illustrates that proton-rich fragments are more readily produced from the more proton-rich emitting sources. These are the typical features associated with "isoscaling" behavior showing that the scaling behavior of the isotope and isotone yield ratios appears naturally within the present CN model framework.

It was reported [29] that the ratio of a certain fragment (N, Z) emission probability from two equilibrated systems can approximately be written as $P_2(N, Z)/P_1(N, Z) \propto exp[(V_1(N, Z) - V_2(N, Z))/T]$, where $V_i(i = 1, 2)$ is of the same meaning as V_{saddle}, namely the interaction energy at the saddle point. An analysis indicates that in the difference $(V_1 - V_2)$, most terms cancel except for terms that are directly related to the isospin of the CN system. These isospin-related terms can be combined as $(\alpha_v\kappa_v - \alpha_s\kappa_s/A^{1/3})\ (N - Z)^2/A = C_{sym}\ (N - Z)^2/A = E_{sym}$. Here $C_{sym}\ (= \alpha_v\kappa_v - \alpha_s\kappa_s/A^{1/3})$ is symmetry energy coefficient [32] incorporating both the volume and surface contributions to the symmetry energy E_{sym} [9]. Hence the value of $(V_1 - V_2)$ is dominantly determined by the difference in symmetry energy between the two emitting sources and their residues. This demonstrates that the origin of the isoscaling phenomenon found in the present frame can be traced to the symmetry energy term in the nuclear binding energy, as also suggested in other work (e.g., [7]).

The isoscaling parameters α and β can be extracted from fits of model predictions to the data points shown in Fig.1. An average value of α is calculated over the range $5 \leq Z \leq 9$, an average β is calculated over the range $5 \leq N \leq 9$. Figure 2 depicts the dependence of the average coefficients α and β *vs.* excitation energy (left panel) and *vs.* inverse of the temperature (right panel). One notices that the absolute values of α and $\mid \beta \mid$ decrease monotonically with excitation energy, implying that isospin effects decrease with increasing excitation energy. There is a significant sensitivity to the excitation energy at low energy, but both the sensitivity to excitation energy and the overall isospin effect are weakened at very high excitation. In addition, it is rather evident that both α and β show a linear dependence on $1/T$.

The surface contribution a_S to the level density parameter a appreciably enhances the fragment emission probability via significant surface entropy effects

[29]. It is therefore interesting to examine the effects of surface entropy on the isoscaling phenomenon in the present model. Results are displayed in Figs.3 and 4, to be compared to Figs.1 and 2 which have been calculated without surface contribution to the entropy. Evidently, familiar isoscaling behavior is predicted well in either case. The linear relationship between isoscaling parameters and inverse temperature is not affected by the surface contribution either, as seen from the right panel of Fig.4. However, an account of the surface term predicts larger values of α and β (comparing Fig.4 and Fig.2), implying that the surface entropy effect on fragment emission becomes stronger with increasing isospin of the emitting sources, also as expected.

Since the scaling behavior of ratios of fragment isotopic yields measured in separate nuclear reactions has been utilized to probe the symmetry energy [7,10,33,32], in the following a symmetry energy coefficient is extracted from the theoretical isoscaling systematically analytical expressions for α and β. It is worth noting that, for a given E^*_{tot}/A these four sources have the same temperature when the surface contribution to the level density parameter is neglected.

It has been shown [9,11] that the isoscaling parameters α and β are related to the symmetry energy coefficient C_{sym} as

$$\alpha = 4\frac{C_{sym}}{T}[(\frac{Z_s}{A_s})^2_1 - (\frac{Z_s}{A_s})^2_2] \tag{9}$$

and

$$\beta = 4\frac{C_{sym}}{T}[(\frac{N_s}{A_s})^2_1 - (\frac{N_s}{A_s})^2_2]. \tag{10}$$

Equations (9) and (10) have been used to constrain the symmetry energy coefficient C_{sym} based on experimental data (e.g. Ref.[10]). Figure 5 depicts the product $\alpha \cdot T$ as a function of $(Z_s/A_s)^2_1 - (Z_s/A_s)^2_2$ and $\beta \cdot T$ as a function of $(N_s/A_s)^2_1 - (N_s/A_s)^2_2$ for the initial four source pairs at various excitation energies. All these systems with different source sizes and isospin asymmetries lie along one single straight line, which illustrates that the isoscaling parameters α and β are not sensitive to the system size. By fitting the theoretical data in Fig. 5 with Eqs. (9) and (10), a symmetry energy coefficient C_{sym} can be obtained to be 27.3 ± 0.1 MeV, which is close to the standard liquid drop model value $C_{sym} = 25$ MeV and also in reasonable agreement with that obtained in Ref.[18]. In that experimental work [18], a value of $C_{sym} = (27.2 \pm 2.2)$ MeV was deduced based on an experimental analysis of 25-MeV/nucleon ^{86}Kr + ^{124,112}Sn reactions. The present analysis demonstrates consistency with other realistic theoretical estimates [34,3] and indicates the validity of an interpretation of isoscaling data in terms of the symmetry energy.

Although the inclusion of the surface term a_S in the level density parameter leads to a difference in the temperatures for the four systems, calculations indicate that for each excitation energy E^*_{tot}/A, the temperature differences among the four sources are less than 0.1 MeV, so the temperature value of four systems can be regarded as approximately constant when the excitation energy per nucleon is fixed. Figure 6 displays the effect of the surface entropy on the extracted symmetry energy coefficient. The value deduced from the theoretical data in Fig. 6 is $C_{sym} = (27.6 \pm 0.1)$ MeV, which is comparable with that predicted without considering the surface entropy effects. The reason is that although the surface entropy increases the values of α and β, it also decreases the value of temperature due to an increased level density parameter a [see Eq. (6)]. Consequently, $\alpha \cdot T$ and $\beta \cdot T$ are little affected by the surface entropy. This result indicates that surface entropy probably has a minor influence on the extraction of the symmetry energy coefficient. Moreover, it also implies that the isoscaling observable is a robust probe of the symmetry energy.

4 SUMMARY AND CONCLUSIONS

In conclusion, in the framework of an extended compound nucleus model, isoscaling behavior and its relation to the nuclear symmetry energy is revealed. The symmetry energy coefficient is found to be $C_{sym} \sim 27$ MeV from an analysis of theoretical "data", suggesting that isoscaling data can be interpreted in terms of the symmetry energy. In addition, in this work surface entropy influences on the isoscaling phenomenon have been studied. The present ECN model approach leaves sufficient room for further improved treatment of interesting physical effects [29], such as nuclear expansion. Such a more detailed analysis is required to study the evolution of the symmetry energy with the excitation energy, as has been explored recently also by Shetty *et al.* [35]. The model will be utilized to deduce the effects of nuclear expansion on the isoscaling parameters and will correspondingly illustrate how to deduce the density dependence of the symmetry energy in such more general and realistic scenarios. Work along this direction is in progress.

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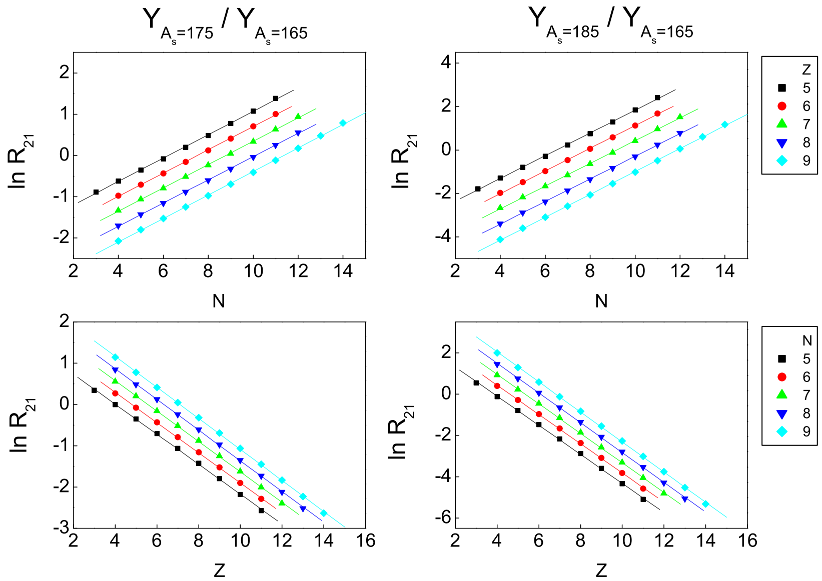

Fig. 1. (Color online) The logarithm of the ratio of elements $Z =$ 5-9 isotopes yields (top panel) and of $N =$ 6-9 isotones yields (bottom panel) from pairing source $A_s = 175$ and $A_s = 165$ (left column) as well as from pairing source $A_s = 185$ and $A_s = 165$ (right column) at excitation energy $E^*_{tot}/A = 5$ MeV/nucleon. Here the calculations do not consider the surface entropy, i.e., not including a_S in the expression for the level density parameter a [see Eq. (6)]. Solid lines are the linear fitting to the data points.

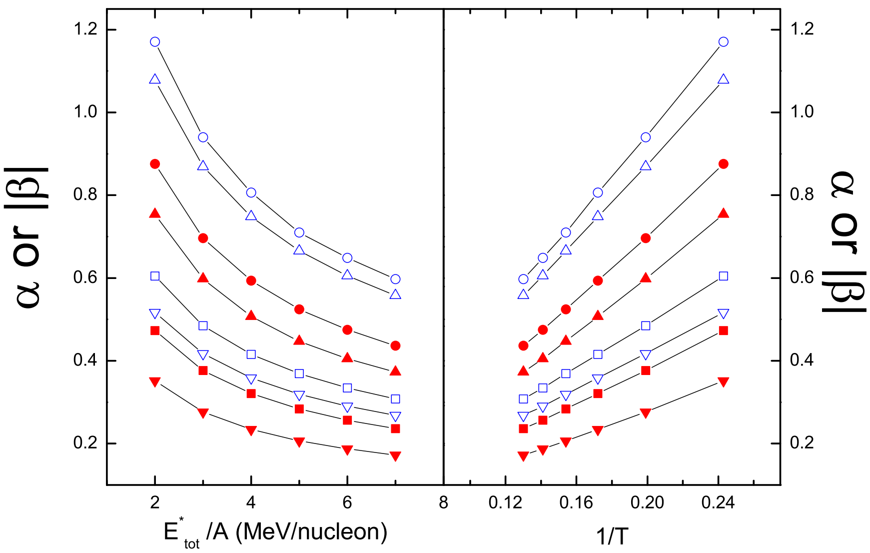

Fig. 2. (Color online) Dependence of isoscaling parameters α and $\mid \beta \mid$ on excitation energy (left panel) and the inverse temperature (right panel) for various source pairs. Symbols in the figures are α (solid symbols) or $\mid \beta \mid$ (open symbols) from four source pairs $Y_{A_s=175}/Y_{A_s=165}$ (squares), $Y_{A_s=185}/Y_{A_s=165}$ (circles), $Y_{A_s=195}/Y_{A_s=175}$ (up-triangles), and $Y_{A_s=195}/Y_{A_s=185}$ (down-triangles). Here the calculations do not consider the surface entropy, i.e., not including a_S in the expression for the level density parameter a [see Eq. (6)].

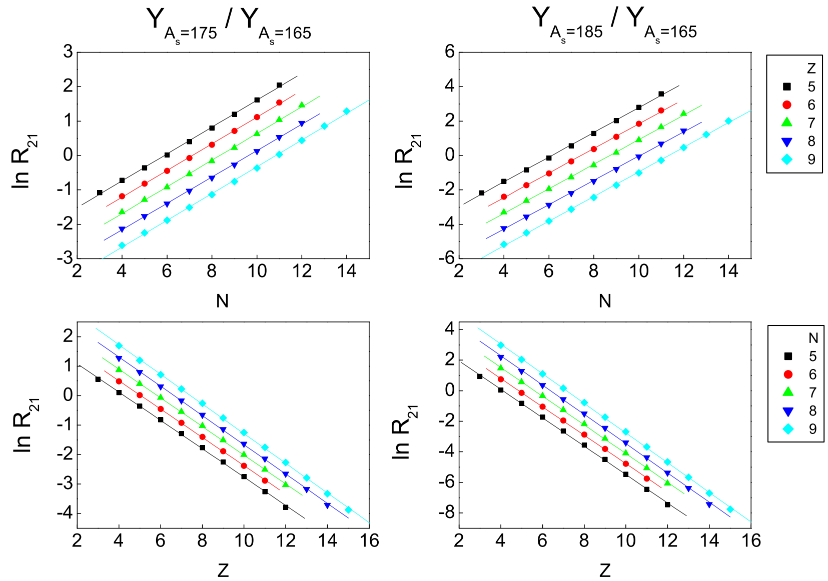

Fig. 3. (Color online) Same as Fig.1 but consider the surface entropy effect via inclusion of a_S in the expression for the level density parameter a [see Eq. (6)].

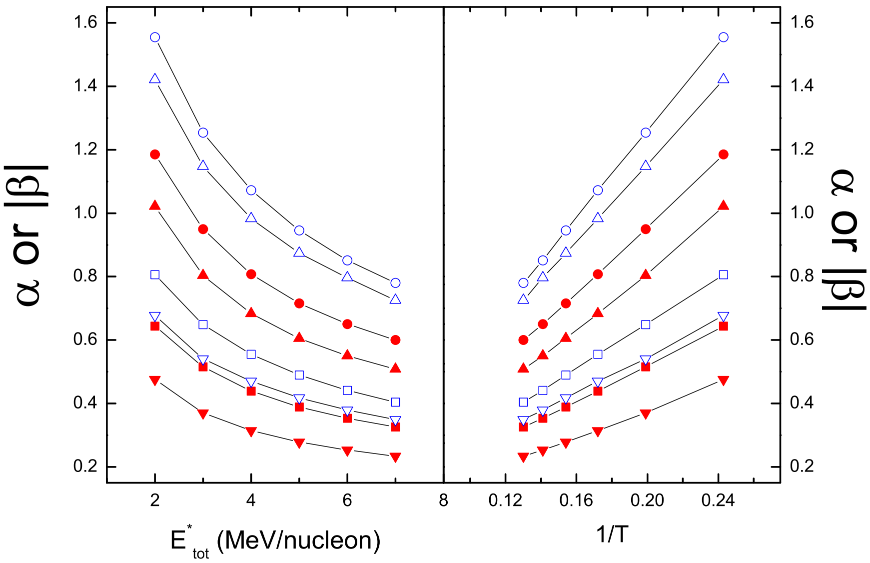

Fig. 4. (Color online) Same as Fig.2 but consider the surface entropy effect via inclusion of a_S in the expression for the level density parameter a [see Eq. (6)].

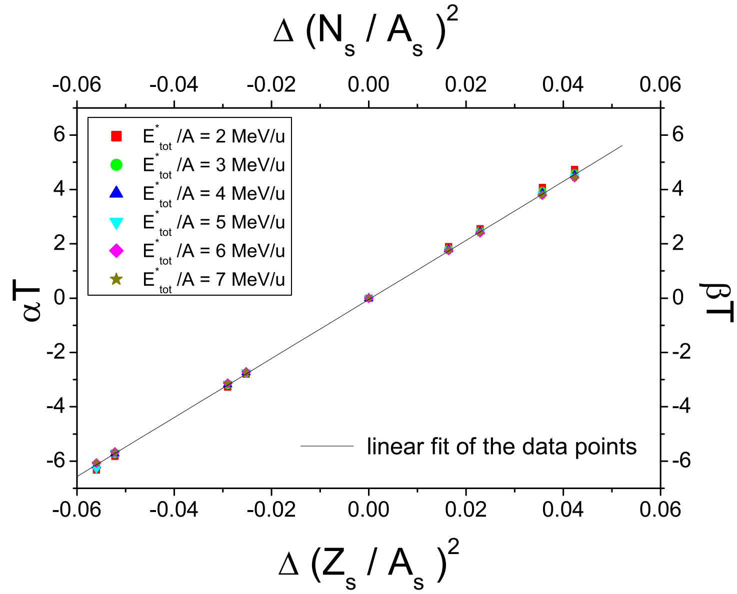

Fig. 5. (Color online) $\alpha \cdot T$ (positive parts) and $\beta \cdot T$ (negative parts) as a function of $(Z_s/A_s)_1^2$ - $(Z_s/A_s)_2^2$ or as a function of $(N_s/A_s)_1^2$ - $(N_s/A_s)_2^2$ of the sources for four source pairs, i.e., $Y_{A_s=175}/Y_{A_s=165}$, $Y_{A_s=185}/Y_{A_s=165}$, $Y_{A_s=195}/Y_{A_s=175}$, and $Y_{A_s=195}/Y_{A_s=185}$, at excitation energies of $E^*_{tot}/A = 2$, 3, 4, 5, 6, and 7 MeV/nucleon. Here the calculations do not consider the surface entropy, i.e., not including a_S in the expression for the level density parameter a [see Eq. (6)].

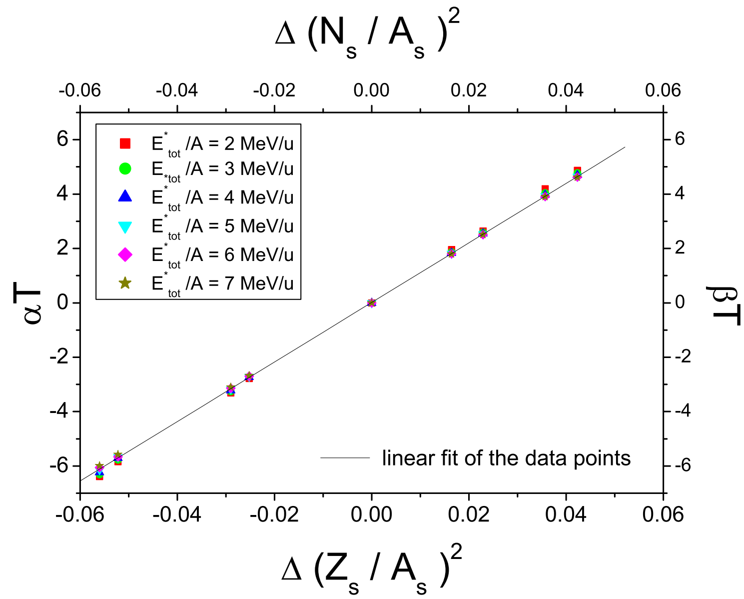

Fig. 6. (Color online) Same as Fig.5 but consider the surface entropy effect via inclusion of a_S in the expression for the level density parameter a [see Eq. (6)].